Solutions: Session 11

Exercise 1

Answers

a) As seen during the course, the relationship between flow-rate and pressure-drop in a pneumotachometer is linear since the nature of flow is laminar. Thus, we have:

$$Q = \frac{\Delta P}{R};\ R = \frac{128\eta L}{\pi D^4};\ Q = 1\frac{L}{s} = 10^{-3}\frac{m^3}{s}$$

Now, for 1 capillary tube:

$$Q_T = \frac{Q}{N_{tubes}} = \frac{10^{-3}}{100} = 10^{-5}\ m^3/s$$

$$R_T = \frac{128 \times 1.8 \times 10^{-5} \times 5 \times 10^{-2}}{\pi \times (10^{-3})^4} = 3.67 \times 10^7\ Pa.\frac{s}{m^3}$$

$$\Delta P = R_T Q_T = 3.67 \times 10^7 \times 10^{-5} = \mathbf{367\ Pa}$$

b) From the condition for laminar flow in a cylindrical tube, we have:

$$N_r = \frac{\rho_{air} v_{max,P} D_P}{\eta_{air}} < 2000 \rightarrow \frac{1.21 \times v_{max,P} \times 20 \times 10^{-3}}{1.8 \times 10^{-5}} < 2000 \rightarrow v_{m,P} < \mathbf{1.48\ m/s}$$

Exercise 2

Answers:

a) Let the received frequency be denoted by f_r. Then, we have:

$$f_d = f_t - f_r = f_t - f_t\left(1 - \frac{v\cos\theta}{c}\right)\left(1 - \frac{v\cos\phi}{c}\right)$$

$$f_d = f_t\left(1 - \left(1 - \frac{v\cos\theta}{c} - \frac{v\cos\phi}{c} + \frac{v^2\cos\theta\cos\phi}{c^2}\right)\right) \approx \frac{v}{c} f_t(\cos\theta + \cos\phi)$$

(since $v^2 \ll c^2$)

$$v = \frac{f_d}{f_t} * \frac{c}{\cos\theta + \cos\phi} = \frac{5.828 \times 10^3}{6 \times 10^6} * \frac{1500}{0.707 + 0.5} = \mathbf{1.21\ m/s}$$

b) Fig. 1 below shows the emitted signal at $t = 0$, and the signals received from the two red blood cells at times t_1 and t_2 (from the closer and farther red blood cells respectively). Their respective Doppler frequencies are indicated on the graph. Note that RBC 2, being deeper inside the artery, suffers greater attenuation than RBC 1, and thus has a smaller signal amplitude. Both signals' amplitudes are smaller than that of the transmitted signal.

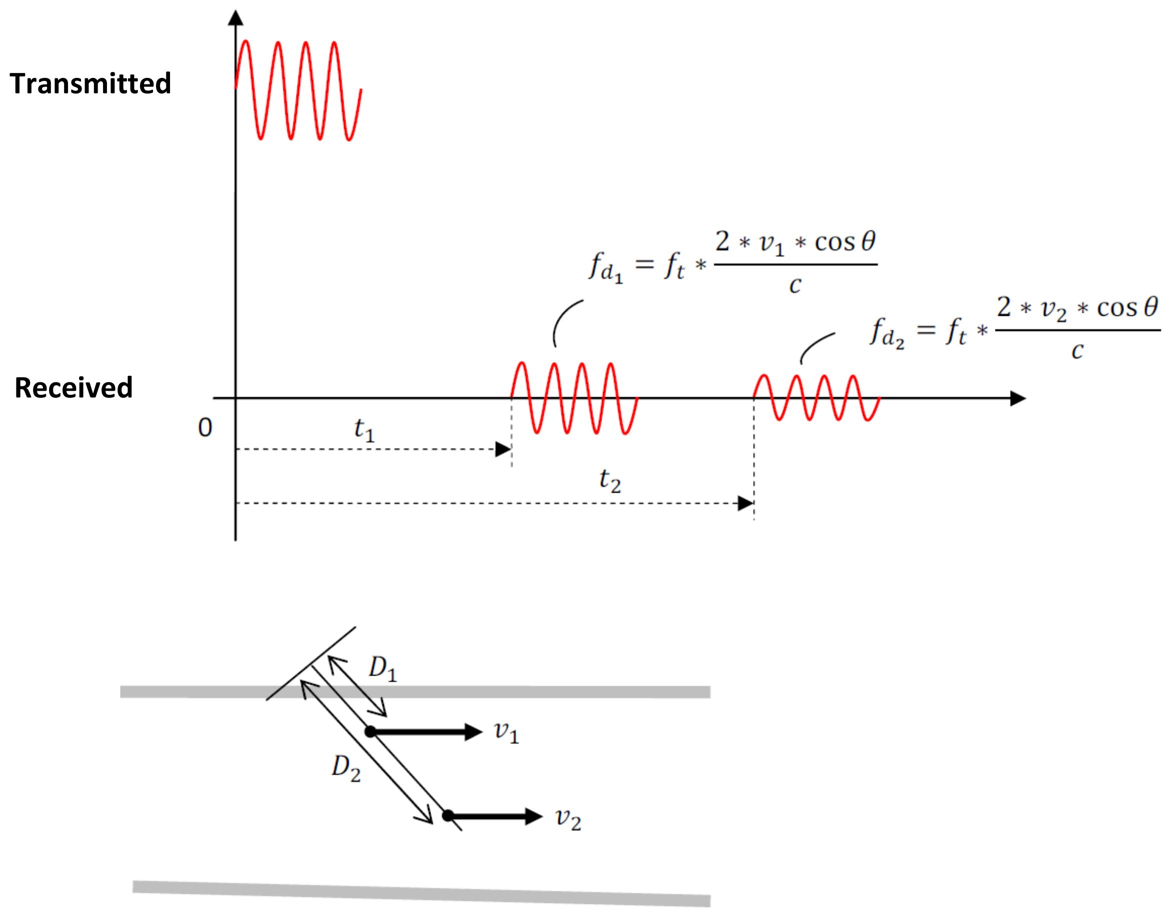

Figure 1

Next, referring to the lower part of Fig. 1, we can calculate the time of arrival (relative to $t = 0$, when the signal was emitted) in terms of the indicated distances D_1 and D_2 as follows):

$$t_1 \approx \frac{2D_1}{c - v_1 cos\theta} \approx \frac{2D_1}{c} \ and \ t_2 \approx \frac{2D_2}{c - v_2 cos\theta} \approx \frac{2D_2}{c}$$

Exercise 3

Answers:

a) Let us first calculate the equivalent resistance and capacitance of the given circuit:

$$R_e = R_S \| R_C \| R_a = 45.45\ M\Omega$$

$$C_e = C_S \| C_C \| C_a = 210\ pF$$

$$U_0(\omega) = \frac{Q}{C_e} * \frac{R_e}{\frac{1}{j\omega C_e} + R_e} = \frac{Q}{C_e} * \frac{j\omega C_e R_e}{1 + j\omega C_e R_e}$$

The term on the right hand side is a high-pass filter with a cut-off frequency of:

$$f_0 = \frac{1}{2\pi R_e C_e} = 16.7\ Hz$$

For frequencies much greater than f_0, the high-pass-filter term simplifies to 1, and we thus obtain:

$$U_0 = \frac{Q}{C_e}$$

From this, the sensitivity can be obtained as follows:

$$S = \frac{U_0}{Q} = \frac{1}{C_e} = 4.76 \times 10^9\ V/C\ = \mathbf{4.76\ mV/pC}$$

b) Multiplying the cable length by 4 is akin to adding 4 single-cable units in the figure provided in the question. Thus, the resultant resistance and capacitance of this longer cable is given by:

$$R_c' = R_c||R_c||R_c||R_c = 25\ M\Omega \qquad C_c' = 400\ pF$$

And the effective resistance and capacitance of the entire circuit can thus be written as:

$$R_e' = R_s||R_c'||R_a = 19.23\ M\Omega \qquad C_e' = 510\ pF$$

This time, the cut-off frequency is equal to:

$$f_0 = \frac{1}{2\pi R_e' C_e'} = 16.2\ Hz$$

Finally, for frequencies much greater than f_0, as before, the sensitivity becomes:

$$S' = \frac{1}{C_e'} = \mathbf{1.96\ mV/pC}$$

Thus, we see that the larger effective capacitance due to the longer cable reduces the sensitivity of this configuration as compared to the shorter cable in part (a).

c) To avoid the issue of the dependence of sensitivity on cable length, we use a charge amplifier. As seen during the course, the circuit for this is as shown in Fig. 2 below. In this configuration, the terminals on either side of the piezo element are connected to the two input terminals of the operational amplifier, and the voltage across the piezo element is almost 0 V. The charge released by the piezo element appears completely on the capacitor C_r. As demonstrated during the course, the sensitivity of this configuration depends only on the value of C_r, and is hence independent of the cable length.

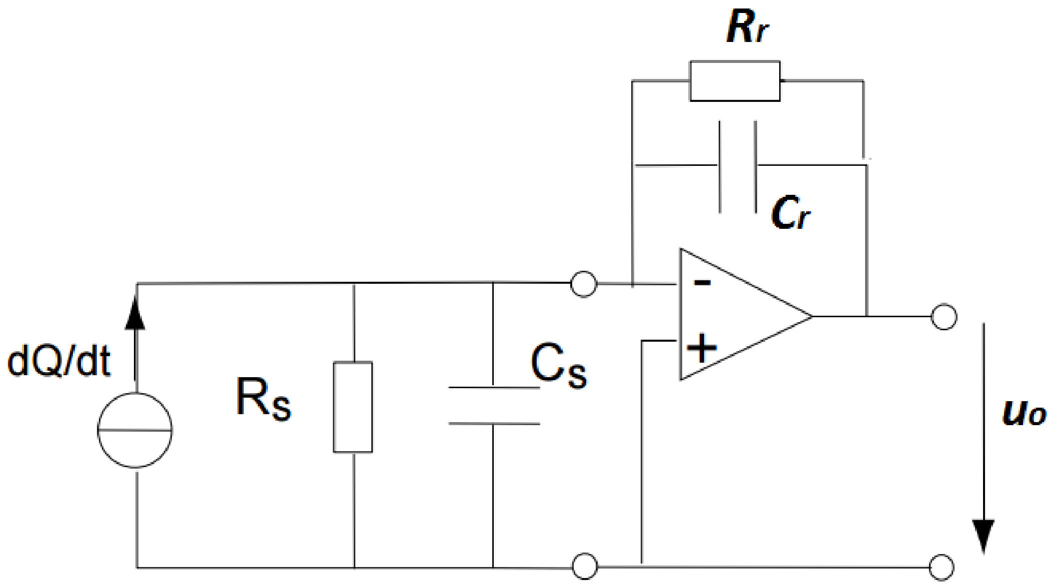

Figure 2

Here, u_o is given by:

$$u_o = -\frac{Q}{C_r} * \frac{j\omega R_e C_e}{1 + j\omega R_e C_e}$$

Just as in the case of the voltage amplifier in parts (a) and (b), this is a high-pass filter with a cut-off frequency of:

$$f_0 = \frac{1}{2\pi C_r R_r} = 1\ Hz\ (given)$$

Thus, we have:

$$C_r R_r = \frac{1}{2\pi f_0} = 0.159$$

We need to now compute sensitivity in units of mV/N. Note that when we express sensitivity as $\boldsymbol{S_c} = \frac{1}{C_r}$, the units are usually mV/pC (as can be seen in parts (a) and (b). When we multiply S_c with the piezoelectric coefficient $\boldsymbol{P}$ of the sensor, having units of pC/N, we obtain the required units of sensitivity as: $\frac{mV}{pC} * \frac{pC}{N} \equiv \frac{mV}{N}$. Thus, denoting the required sensitivity of $10\ mV/N$ as S_T, we have:

$$S_T = S_c \times P = S_c \times 2\ pC/N = 10\ mV/N$$

Thus, we obtain $S_c = 5\ mV/pC$, from which we find the values of the resistance and capacitance as:

$$\boldsymbol{C_r = 200\ pF;\ R_r = 796\ M\Omega}$$

d) As we have seen several times before, with one gauge on a Wheatstone bridge with three other resistances having the same value as the resistance of the gauge when there is no deformation, the measured voltage is:

$$U_m = \frac{1}{4} K \varepsilon \mathrm{U_{supply}}$$

Let S_M denote the sensitivity of this bridge in terms of $\frac{deformation}{force}$). Then, $S_M = \varepsilon / F$. Thus, the above equation for U_m can be rewritten as:

$$U_m = \frac{1}{4} K S_M F U_{supply}$$

$$S_M = \frac{4\left(\frac{U_m}{F}\right)}{K U_{supply}} = \frac{4 \times \left(10 \times 10^{-3} \frac{V}{N}\right)}{150 \times (10\ V)} = \mathbf{2.67 \times 10^{-5}\ N^{-1}}$$